Primitives par parties : énoncés

$$\int f(x).g'(x).dx = f(x).g(x) - \int f'(x).g(x).dx$$

1. $\int e^{x}.\cos x.dx$
2. $\int x^{4}.\ln x.dx$
3. $\int e^{x}.\sin x.dx$
4. $\int e^{-3x}.\cos x.dx$
5. $\int e^{-5x}.\sin x.dx$
6. $\int e^{3x}.\cos 2x.dx$
7. $\int e^{2x}.\cos 3x.dx$
8. $\int (\arcsin x)^{2} dx$
9. $\int x^{2}.\ln(3x).dx$
10. $\int x^{3} \ln x.dx$
11. $\int x^{2} e^{3x} dx$
12. $\int \arccos x \, dx$
13. $\int x^{5} \, \ln 2x \, dx$

Primitives par parties : corrigés

$$\int f(x).g'(x).dx = f(x).g(x) - \int f'(x).g(x).dx$$

1. $P = \int e^x.\cos x.dx$

- $f = e^x$ $f' = e^x$

  $g' = \cos x$ $g = \sin x$ $P = e^x.\sin x - \underbrace{\int e^x.\sin x.dx}_{P_1}$

- $f = e^x$ $f' = e^x$

  $g' = \sin x$ $g = -\cos x$ $P_1 = e^x.(-\cos x) + \underbrace{\int e^x.\cos x.dx}_{P}$

$$P = e^x.\sin x + e^x.\cos x - P \Leftrightarrow P = \frac{e^x(\sin x + \cos x)}{2} + C$$

2. $P = \int x^4.\ln x.dx$

$f = \ln x$ $f' = \dfrac{1}{x}$

$g' = x^4$ $g = \dfrac{x^5}{5}$

$$P = \frac{x^5}{5}.\ln x - \int \frac{1}{x}.\frac{x^5}{5}.dx = \frac{x^5}{5}.\ln x - \int \frac{x^4}{5}.dx = \frac{x^5}{5}.\ln x - \frac{x^5}{25} + C$$

3. $P = \int e^x.\sin x.dx$

- $f = e^x$ $f' = e^x$

  $g' = \sin x$ $g = -\cos x$ $P = e^x.(-\cos x) + \underbrace{\int e^x.\cos x.dx}_{P_1}$

- $f = e^x$ $f' = e^x$

  $g' = \cos x$ $g = \sin x$ $P_1 = e^x.\sin x - \underbrace{\int e^x.\sin x.dx}_{P}$

$$P = e^x.(-\cos x) + e^x.\sin x - P \Leftrightarrow P = \frac{e^x(\sin x - \cos x)}{2} + C$$

4. $P = \int e^{-3x}.\cos x.dx$

- f(x) = e^{-3x} f'(x) = -3 e^{-3x}

  g'(x) = cosx g(x) = sinx $P = e^{-3x}.\sin x + 3.\underbrace{\int e^{-3x}.\sin x.dx}_{P_1}$

- f(x) = e^{-3x} f'(x) = -3 e^{-3x}

  g'(x) = sinx g(x) = -cosx $P_1 = e^{-3x}.(-\cos x) - 3\underbrace{\int e^{-3x}.\cos x.dx}_{P}$

$$P = e^{-3x}.\sin x + 3.\left(e^{-3x}(-\cos x) - 3P\right)$$
$$\Leftrightarrow 10.P = e^{-3x}.\sin x - 3.e^{-3x}.\cos x$$
$$\Leftrightarrow P = \frac{e^{-3x}(\sin x - 3\cos x)}{10} + C$$

5. $P = \int e^{-5x}.\sin x.dx$

- f(x) = e^{-5x} f'(x) = -5. e^{-5x}

  g'(x) = sinx g(x) = -cosx $P = -e^{-5x}.\cos x - 5.\underbrace{\int e^{-5x}.\cos x.dx}_{P_1}$

- f(x) = e^{-5x} f'(x) = -5 e^{-5x}

  g'(x) = cosx g(x) = sinx $P_1 = e^{-5x}.\sin x + 5\underbrace{\int e^{-5x}.\sin x.dx}_{P}$

$$P = -e^{-5x}.\cos x - 5.\left(e^{-5x}.\sin x + 5P\right)$$
$$\Leftrightarrow 26.P = -e^{-5x}.\cos x - 5.e^{-5x}.\sin x$$
$$\Leftrightarrow P = \frac{-e^{-5x}(\cos x + 5.\sin x)}{26} + C$$

6. $P = \int e^{3x}.\cos 2x.dx$

➢ $\begin{array}{ll} f = e^{3x} & f' = 3.e^{3x} \\ g' = \cos 2x & g = \dfrac{\sin 2x}{2} \end{array}$ $\quad P = e^{3x}.\dfrac{\sin 2x}{2} - \dfrac{3}{2}\underbrace{\int e^{3x}.\sin 2x.dx}_{P_1}$

➢ $\begin{array}{ll} f = e^{3x} & f' = 3.e^{3x} \\ g' = \sin 2x & g = \dfrac{-\cos 2x}{2} \end{array}$ $\quad P_1 = e^{3x}.\dfrac{-\cos 2x}{2} + \dfrac{3}{2}\underbrace{\int e^{3x}.\cos 2x.dx}_{P}$

$$P = e^{3x}.\frac{\sin 2x}{2} - \frac{3}{2}\left(e^{3x}.\frac{-\cos 2x}{2} + \frac{3}{2}P\right) \Leftrightarrow P = e^{3x}.\frac{\sin 2x}{2} + \frac{3}{4}e^{3x}.\cos 2x - \frac{9}{4}P$$

$$\Leftrightarrow \frac{13}{4}P = e^{3x}.\frac{\sin 2x}{2} + \frac{3}{4}e^{3x}.\cos 2x \Leftrightarrow P = \frac{2.e^{3x}.\sin 2x}{13} + \frac{3.e^{3x}.\cos 2x}{13} + C$$

$$\Leftrightarrow P = \frac{e^{3x}(2.\sin 2x + 3.\cos 2x)}{13} + C$$

7. $P = \int e^{2x}.\cos 3x.dx$

➢ $\begin{array}{ll} f = e^{2x} & f' = 2.e^{2x} \\ g' = \cos 3x & g = \dfrac{\sin 3x}{3} \end{array}$ $\quad P = e^{2x}.\dfrac{\sin 3x}{3} - \dfrac{2}{3}\underbrace{\int e^{2x}.\sin 3x.dx}_{P_1}$

➢ $\begin{array}{ll} f = e^{2x} & f' = 2.e^{2x} \\ g' = \sin 3x & g = \dfrac{-\cos 3x}{3} \end{array}$ $\quad P_1 = e^{2x}.\dfrac{-\cos 3x}{3} + \dfrac{2}{3}\underbrace{\int e^{2x}.\cos 3x.dx}_{P}$

$$P = e^{2x}.\frac{\sin 3x}{3} - \frac{2}{3}\left(e^{2x}.\frac{-\cos 3x}{3} + \frac{2}{3}P\right) \Leftrightarrow P = e^{2x}.\frac{\sin 3x}{3} + \frac{2}{9}e^{2x}.\cos 3x - \frac{4}{9}P$$

$$\Leftrightarrow \frac{13}{9}P = e^{2x}.\frac{\sin 3x}{3} + \frac{2}{9}e^{2x}.\cos 3x \Leftrightarrow P = \frac{3.e^{2x}.\sin 3x}{13} + \frac{2.e^{2x}.\cos 3x}{13} + C$$

$$\Leftrightarrow P = \frac{e^{2x}(3.\sin 3x + 2.\cos 3x)}{13} + C$$

8. $\mathbf{P = \int (\arcsin x)^2 dx}$

- $f(x) = (\arcsin x)^2$ $\qquad f'(x) = 2.\arcsin x.\dfrac{1}{\sqrt{1-x^2}}$

  $g'(x) = 1$ $\qquad g(x) = x$

$$\mathbf{P = x.(\arcsin x)^2 - \underbrace{2\int \arcsin x.\frac{x}{\sqrt{1-x^2}}.dx}_{P_1}}$$

- $f(x) = \arcsin x$ $\qquad f'(x) = \dfrac{1}{\sqrt{1-x^2}}$

  $g'(x) = \dfrac{x}{\sqrt{1-x^2}}$ $\qquad g(x) = -\sqrt{1-x^2}$ $\qquad$ (poser $t = 1-x^2$)

$$\mathbf{P_1 = -\sqrt{1-x^2}.\arcsin x + \int 1.dx = -\sqrt{1-x^2}.\arcsin x + x}$$

$$\mathbf{P = x.(\arcsin x)^2 - 2\left(-\sqrt{1-x^2}.\arcsin x + x\right) + C}$$
$$\mathbf{= x.(\arcsin x)^2 + 2\sqrt{1-x^2}.\arcsin x - 2x + C}$$

9. $\mathbf{P = \int x^2.\ln(3x).dx}$

$f = \ln(3x)$ $\qquad f' = \dfrac{3}{3x} = \dfrac{1}{x}$

$g' = x^2$ $\qquad g = \dfrac{x^3}{3}$

$$\mathbf{P} = \frac{x^3.\ln(3x)}{3} - \int \frac{1}{x}.\frac{x^3}{3}.dx = \frac{x^3.\ln(3x)}{3} - \frac{1}{3}\int x^2.dx$$

$$\mathbf{= \frac{x^3.\ln(3x)}{3} - \frac{x^3}{9} + C}$$

10. $\mathbf{P = \int x^3 \ln x \, dx}$

$f = \ln x \qquad f' = \frac{1}{x}$

$g' = x^3 \qquad g = \frac{x^4}{4}$

$$\mathbf{P =} \frac{x^4}{4}\ln x - \int \frac{1}{x}\frac{x^4}{4}dx = \frac{x^4}{4}\ln x - \frac{1}{4}\int x^3 dx = \frac{x^4}{4}\ln x - \frac{1}{4}\frac{x^4}{4} + C$$

$$\mathbf{= \frac{x^4}{4}\ln x - \frac{x^4}{16} + C}$$

11. $\mathbf{P = \int x^2 e^{3x} \, dx}$

$f = x^2 \qquad f' = 2x$

$g' = e^{3x} \qquad g = \frac{e^{3x}}{3}$

$$P = x^2\frac{e^{3x}}{3} - \int 2x\frac{e^{3x}}{3}dx$$

$$\mathbf{P = x^2\frac{e^{3x}}{3} - \frac{2}{3}\underbrace{\int x e^{3x} dx}_{P_1}}$$

$f = x \qquad f' = 1$

$g' = e^{3x} \qquad g = \frac{e^{3x}}{3}$

$$P_1 = x\frac{e^{3x}}{3} - \int 1.\frac{e^{3x}}{3}dx = x\frac{e^{3x}}{3} - \frac{1}{3}\int e^{3x}dx$$

$$\mathbf{P_1 = x\frac{e^{3x}}{3} - \frac{e^{3x}}{9}}$$

$$\mathbf{P = x^2\frac{e^{3x}}{3} - \frac{2}{3}\left(x\frac{e^{3x}}{3} - \frac{e^{3x}}{9}\right) + C}$$

$$\mathbf{= x^2\frac{e^{3x}}{3} - \frac{2}{9}x e^{3x} + \frac{2}{27}e^{3x} + C}$$

$$\mathbf{= \frac{e^{3x}}{27}\left(9x^2 - 6x + 2\right) + C}$$

12. $\mathbf{P = \int \arccos x \, dx}$

$$P = \int 1.\arccos x \, dx$$

- $f = \arccos x \qquad f' = \dfrac{-1}{\sqrt{1-x^2}}$

  $g' = 1 \qquad g = x$

$$P = x \arccos x - \int \frac{-1}{\sqrt{1-x^2}} \, x \, dx$$

$$\mathbf{P = x \arccos x + \underbrace{\int \frac{x}{\sqrt{1-x^2}} dx}_{P_1}}$$

- Posons : $t = 1 - x^2$

$$dt = -2x \, dx \quad \Leftrightarrow \quad dx = \frac{dt}{-2x}$$

$$P_1 = \int \frac{x}{\sqrt{t}} \frac{dt}{-2x} = \frac{-1}{2} \int t^{-1/2} dt = \frac{-1}{2} \frac{t^{1/2}}{\frac{1}{2}} + C$$

$$\mathbf{P_1 = -\sqrt{t} = -\sqrt{1-x^2}}$$

$$\mathbf{P = x \arccos x - \sqrt{1-x^2} + C}$$

13. $\mathbf{P = \int x^5 \ln 2x \, dx}$

$f = \ln 2x \qquad f' = \dfrac{1}{x}$

$g' = x^5 \qquad g = \dfrac{x^6}{6}$

$$P = \frac{x^6}{6} \ln 2x - \int \frac{1}{x} \frac{x^6}{6} dx = \frac{x^6}{6} \ln 2x - \frac{1}{6} \int x^5 \, dx = \frac{x^6}{6} \ln 2x - \frac{1}{6} \frac{x^6}{6} + C$$

$$P = \frac{x^6}{6} \ln 2x - \frac{x^6}{36} + C = \frac{x^6}{36} \left(6 \ln 2x - 1\right) + C$$