Factorisations

§1. Mise en évidence

a) Exemple :

$$5a^2b^2 - 2a^3b = ab(5ab - 2a^2)$$

b) Exercices :

1) Factoriser le plus loin possible:

a) $6a^2b-2a^2b^3+8ab^2 = 2ab(3a - ab^2 + 4b)$

b) $2a(x-y)-3b(x-y) = (x - y)(2a - 3b)$

c) $(a-b)(x-2)+3x(b-a)$
$= (a - b)(x - 2) - 3x(a - b) = (a - b)(x - 2 - 3x) = (a - b)(-2x - 2)$
$= -2(a - b)(x + 1)$

d) $(x+y)(3a+2)-(x+y) = (x + y)(3a + 2 - 1) = (x + y)(3a + 1)$

e) $(a+1)^2+2(a+1) = (a + 1)(a + 1 + 2) = (a + 1)(a + 3)$

f) $3(2-x)^2-3(x-2)^3 = 3(x - 2)^2 - 3(x - 2)^3 = 3(x - 2)^2[1 - (x - 2)]$
$= 3(x - 2)^2(1 - x + 2) = 3(x - 2)^2(3 - x)$

§2. Produits remarquables

a) Formules :

$$\boxed{\begin{array}{c} \mathbf{a^2 - b^2 = (a - b)(a + b)} \\ \mathbf{a^2 \pm 2ab + b^2 = (a \pm b)^2} \\ \mathbf{a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)} \\ \mathbf{a^3 \pm 3a^2b + 3ab^2 \pm b^3 = (a \pm b)^3} \end{array}}$$

b) Exercices :

2) Factoriser le plus loin possible:

a) $a^2 - 9b^2 = (a-3b)(a+3b)$

b) $x^4 - y^6 = (x^2 - y^3)(x^2 + y^3)$

c) $-16a^2 + 9b^4 = 9b^4 - 16a^2 = (3b^2 - 4a)(3b^2 + 4a)$

d) $(a - b)^2 - a^2 = a^2 - 2ab + b^2 - a^2 = b^2 - 2ab = b(b - 2a)$

e) $(a - b)^3 - (a - b) = (a-b)[(a-b)^2 - 1] = (a-b)(a-b-1)(a-b-1)$

f) $(a - 2b)^2 - (2a + 5b)^2 = [(a-2b) - (2a+5b)][(a-2b) + (2a+5b)]$
$= (a - 2b - 2a - 5b)(a - 2b + 2a + 5b) = (-a - 7b)(3a + 3b)$
$= -3(a + 7b)(a + b)$

g) $4(a + b)^2 - 9b^2 = [2(a+b) - 3b][2(a+b) + 3b] = (2a + 2b - 3b)(2a + 2b + 3b)$
$= (2a - b)(2a + 5b)$

h) $\frac{3}{5}x^2 - \frac{5}{3}y^2 = \frac{9}{15}x^2 - \frac{25}{15}y^2 = \frac{1}{15}(9x^2 - 25y^2) = \frac{1}{15}(3x - 5y)(3x + 5y)$

i) $a^2 + 4a + 4 = (a+2)^2$

j) $a^4 + a^2 + \frac{1}{4} = \left(a^2 + \frac{1}{2}\right)^2$

k) $8 - 12a + 6a^2 - a^3 = (2 - a)^3$

l) $a^3 + a^2b + \frac{ab^2}{3} + \frac{b^3}{27} = \left(a + \frac{b}{3}\right)^3$

m) $8a^6 + 36a^4b + 27b^3 + 54a^2b^2 = (2a + 3b)^3$

n) $16a^4 + 2a + 24a^3 + 12a^2 = 2a(8a^3 + 1 + 12a^2 + 6a) = 2a(2a + 1)^3$

o) $a^7 - 3a^5 + 3a^3 - a = a(a^6 - 3a^4 + 3a^2 - 1) = a(a^2 - 1)^3 = a(a-1)^3(a+1)^3$

p) $8x^3-1 = (2x-1)(4x^2+2x+1)$

q) $x - x^4 = x(1-x^3) = x(1-x)(1+x+x^2)$

r) $(x - y)^3 - y^3 = (x-y-y)[(x-y)^2+(x-y)y+y^2]$
$= (x-2y)(x^2-2xy+y^2+xy-y^2+y^2) = (x-2y)(x^2-xy+y^2)+$

s) $1 - 8(a + b)^3 = [1-2(a+b)][1+2(a+b)+4(a+b)^2]$
$= (1-2a-2b)(1+2a+2b+4a^2+8ab+4b^2)$

t) $x^3 + \dfrac{1}{27} = \left(x+\frac{1}{3}\right)\left(x^2-\frac{x}{3}+\frac{1}{9}\right)$

§3. Groupements

a) Exemple :

$$ax + bx + ay + by = x(a + b) + y(a + b) = (a + b)(x + y)$$

b) Exercices :

3) Factoriser le plus loin possible:

a) $x^3 + 2x^2 - x - 2 = x^2(x + 2) - (x + 2) = (x + 2)(x^2 - 1) = (x + 2)(x - 1)(x + 1)$

b) $a^2 - 2ab - 1 + b^2 = (a - b)^2 - 1 = (a - b - 1)(a - b + 1)$

§4. Méthode des diviseurs binômes

a) Exemple :

$$p(x) = x^3 + 4x^2 + 5x + 6$$

Considérer les diviseurs (entiers positifs et négatifs) du terme indépendant (= 6) du polynôme p(x) : $D_6 = \{\pm 1, \pm 2, \pm 3, \pm 6\}$.

Calculer la valeur de p(x) pour ces différentes valeurs jusqu'au moment où l'on obtient zéro.

☺ : tous les coefficients étant positifs, il est inutile de tester les diviseurs entiers positifs (pourquoi ?)

- *Méthode p-c (papier-crayon)*

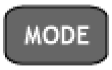
$p(-1) = (-1)^3 + 4(-1)^2 + 5(-1) + 6 = -1 + 4 - 5 + 6 = 4 \neq 0$

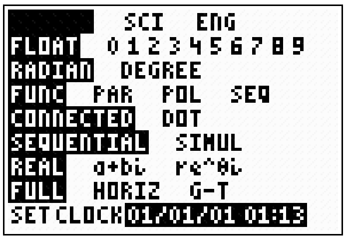
$p(-2) = (-2)^3 + 4(-2)^2 + 5(-2) + 6 = -8 + 16 - 10 + 6 = 4 \neq 0$

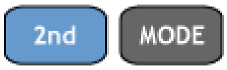
$p(-3) = (-3)^3 + 4(-3)^2 + 5(-3) + 6 = -27 + 36 - 15 + 6 = 0 \Leftrightarrow$ p(x) est divisible par **x + 3**

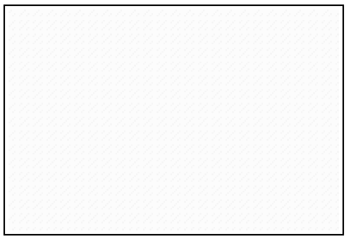
- *Calculatrice graphique*

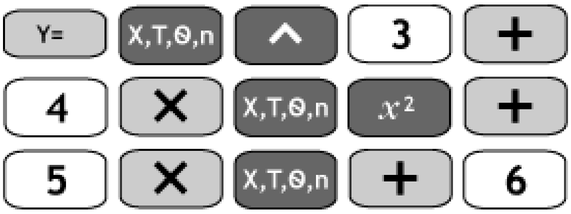
  ❖ vérifier les paramètres du mode :

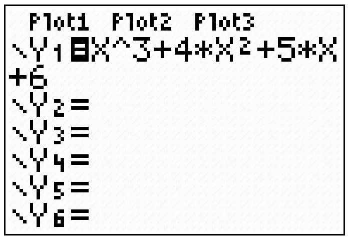


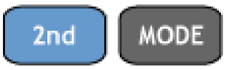


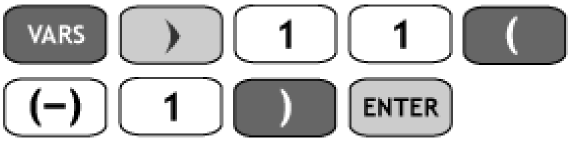


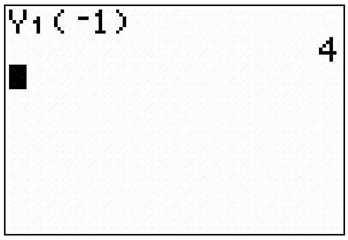


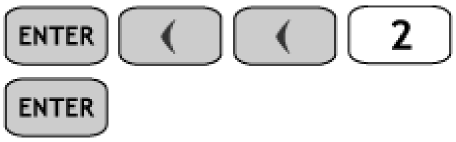


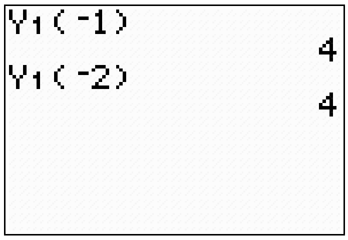


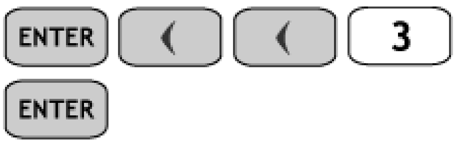


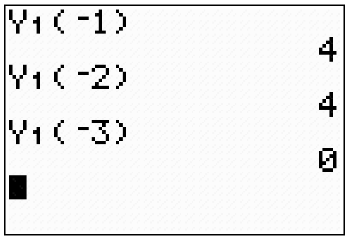












- *Règle de Horner*

	1	4	5	6
-3		- 3	- 3	- 6
	1	1	2	0

Le quotient de la division de p(x) par (x + 3) est : $x^2 + x + 2$ et donc :

$$p(x) = x^3 + 4x^2 + 5x + 6 = (x+3)(x^2 + x + 2)$$

b) <u>Exercices</u> :

4) Simplifier les fractions suivantes :

a) $\dfrac{2x^3+9x^2-32x+21}{4x^3+25x^2-22x-7} = \dfrac{(x-1)(x+7)(2x-3)}{(x-1)(x+7)(4x+1)} = \dfrac{2x-3}{4x+1}$

b) $\dfrac{x^4-5x^2+4}{6-5x+x^3-2x^2} = \dfrac{(x-2)(x-1)(x+1)(x+2)}{(x-3)(x-1)(x+2)} = \dfrac{(x-2)(x+1)}{x-3}$

§5. Exercices récapitulatifs

5) Factoriser le plus loin possible:

a) $x(a - b) + 3(b - a) = (a-b)(x-3)$

b) $a^4 + b^4 - 2a^2b^2 = (a-b)^2(a+b)^2$

c) $(-a - b)^3 + 4(a + b) = (a+b)(2-a-b)(2+a+b)$

d) $a^3 + a^2 + 1 + a = (a+1)(a^2+1)$

e) $2a^3 - 6a^2 + 6a - 2 = 2(a-1)^3$

f) $a(a + c) - b(b + c) = (a-b)(a+b+c)$

g) $a^6 - b^6 = (a-b)(a^2+ab+b^2)(a+b)(a^2-ab+b^2)$

h) $a^8 - b^8 = (a-b)(a+b)(a^2+b^2)(a^4+b^4)$

i) $a^3 - b^3 - a^2 + b^2 = (a-b)(a^2+ab+b^2-a-b)$

j) $a^2b + 2ab + b = b(a+1)^2$

k) $m^2p^2 - p^2n^2 - m^2q^2 + n^2q^2 = (m-n)(m+n)(p-q)(p+q)$

l) $a^2 + 1 - b^2 - 2a = (a-1-b)(a-1+b)$

m) $(a^2 + b^2)^2 - 4a^2b^2 = (a-b)^2(a+b)^2$

n) $(a - b)^3 - (a^3 - b^3) = -3ab(a-b)$

o) $x^5 + 4 - 4x^3 - x^2 = (x-2)(x+2)(x-1)(x^2+x+1)$

p) $4a^2 - 9b^2 - c^2 + 6bc = (2a-3b+c)(2a+3b-c)$

q) $(a - b)^2 + 4ab = (a+b)^2$

r) $c(a^2 - c) + a(a - c^2) = (a-c)(ac+a+c)$

6) Résoudre les équations suivantes :

a) $3x^3 - 22x^2 + 37x - 10 = 0$

$\Leftrightarrow (x-5)(x-2)(3x-1) = 0$

$\Leftrightarrow x = 5 \text{ ou } x = 2 \text{ ou } x = \frac{1}{3}$

- $S = \left\{\frac{1}{3}, 2\ 5\right\}$

b) $4x^4 - 35x^3 + 108x^2 - 135x + 54 = 0$

$\Leftrightarrow (x-3)^2(x-2)(4x-3) = 0$

$\Leftrightarrow x = 3 \text{ ou } x = 2 \text{ ou } x = \frac{3}{4}$

- $S = \left\{\frac{3}{4}, 2, 3\right\}$