Primitives immédiates : énoncés

$$\int a.dx = a.x + C \qquad a \in R$$

$$\int x^m.dx = \frac{x^{m+1}}{m+1} + C \qquad m \neq -1$$

$$\int \cos x.dx = \sin x + C$$

$$\int \sin x.dx = -\cos x + C$$

$$\int \frac{1}{\cos^2 x} dx = \int \left(1 + tg^2 x\right).dx = tgx + C$$

$$\int \frac{1}{\sin^2 x} dx = \int \left(1 + cotg^2 x\right).dx = -cotg\, x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C = -\arccos x + C$$

$$\int \frac{1}{1+x^2} dx = arctgx + C = -arccotg\, x + C$$

$$\int (f(x) + g(x)).dx = \int f(x).dx + \int g(x).dx$$

$$\forall r \in R_0 : \int r.f(x).dx = r.\int f(x).dx$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int e^x.dx = e^x + C$$

1. $\int \left(3x^2 - 2x + 5\right)dx$

2. $\int \left(\cos 5x + \frac{2}{\cos^2 x}\right)dx$

3. $\int \frac{1}{(3x-1)^2} dx$

4. $\int \left(3x^2 + 5x - 4\right)dx$

5. $\int \left(\sin 2x + \frac{5}{\sin^2 x}\right)dx$

6. $\int \frac{1}{(5x-2)^3} dx$

7. $\int (2x+3)^3 dx$

8. $\int \sin(3x+5)dx$

9. $\int (3x-1)^6 dx$

10. $\int \frac{4}{x^5} dx$

11. $\int \sin(7x)dx$

12. $\int x^2\sqrt{x}.dx$

13. $\int \sqrt[3]{x^2}.dx$

14. $\int \cos\left(3x-\frac{\pi}{6}\right)dx$

15. $\int \frac{4x^2+1}{2x^2} dx$

16. $\int (3x+1)^7 dx$

17. $\int \cos(9x)dx$

18. $\int \left(1+tg^2x\right)dx$

19. $\int 2\sin\frac{x}{5} dx$

20. $\int \frac{3x^2+2x-1}{x-1} dx$

21. $\int (3x-7)^6 dx$

22. $\int \frac{x^3-3x^2+1}{x^3} dx$

23. $\int \frac{2x^2+x+4}{2x-1} dx$

24. $\int \frac{\sqrt[3]{x^4}.dx}{3}$

25. $\int e^{\pi.r}\left(r^3+7.r+\frac{14}{r^7}\right).dx$

Primitives immédiates : corrigés

$$\int a.dx = a.x + C \qquad a \in R$$
$$\int x^m.dx = \frac{x^{m+1}}{m+1} + C \qquad m \neq -1$$
$$\int \cos x.dx = \sin x + C$$
$$\int \sin x.dx = -\cos x + C$$
$$\int \frac{1}{\cos^2 x}dx = \int \left(1 + tg^2 x\right).dx = tgx + C$$
$$\int \frac{1}{\sin^2 x}dx = \int \left(1 + cotg^2 x\right).dx = -cotg\, x + C$$
$$\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin x + C = -\arccos x + C$$
$$\int \frac{1}{1+x^2}dx = arctgx + C = -arccotg\, x + C$$
$$\int (f(x) + g(x)).dx = \int f(x).dx + \int g(x).dx$$
$$\forall r \in R_0 : \int r.f(x).dx = r.\int f(x).dx$$
$$\int \frac{1}{x}dx = \ln|x| + C$$
$$\int e^x.dx = e^x + C$$

1. $\int \left(3x^2 - 2x + 5\right)dx$

$$= 3\frac{x^3}{3} - 2\frac{x^2}{2} + 5x + C = x^3 - x^2 + 5x + C$$

2. $\int \left(\cos 5x + \frac{2}{\cos^2 x}\right)dx$

$$= \frac{\sin(5x)}{5} + 2.tgx + C$$

3. $\int \frac{1}{(3x-1)^2}dx = \int (3x-1)^{-2}dx$

$$= \frac{(3x-1)^{-1}}{-1.3} + C = -\frac{1}{3(3x-1)} + C$$

4. $\int \left(3x^2 + 5x - 4\right)dx$

$= 3\frac{x^4}{4} + 5\frac{x^2}{2} - 4x + C$

5. $\int \left(\sin 2x + \frac{5}{\sin^2 x}\right)dx$

$= -\frac{\cos(2x)}{2} - 5\cot gx + C$

6. $\int \frac{1}{(5x-2)^3}dx = \int (5x-2)^{-3}dx$

$= \frac{(5x-2)^{-2}}{-2.5} + C = \frac{-1}{10(5x-2)^2} + C$

7. $\int (2x+3)^3 dx$

$= \frac{(2x+3)^4}{8} + C$

8. $\int \sin(3x+5)dx$

$= \frac{-\cos(3x+5)}{3} + C$

9. $\int (3x-1)^6 dx$

$= \frac{(3x-1)^7}{21} + C$

10. $\int \frac{4}{x^5}dx$

$= 4\frac{x^{-5+1}}{-5+1} + C = \frac{-1}{x^4} + C$

11. $\int \sin(7x)dx$

$= \frac{-\cos(7x)}{7}dx$

12. $\int x^2\sqrt{x}.dx$

$$= \int x^{\frac{5}{2}}.dx = \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + C = \frac{2}{7}x^3\sqrt{x} + C$$

13. $\int \sqrt[3]{x^2}.dx$

$$= \int x^{\frac{2}{3}}.dx = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + C = \frac{3}{5}x\sqrt[3]{x^2} + C$$

14. $\int \cos\left(3x - \frac{\pi}{6}\right).dx$

$$= -\frac{\sin\left(3x - \frac{\pi}{6}\right)}{3} + C$$

15. $\int \frac{4x^2+1}{2x^2}dx$

$$= \int\left(2 + \frac{1}{2}x^{-2}\right)dx = 2x + \frac{1}{2}\frac{x^{-1}}{-1} + C = 2x - \frac{1}{2x} + C$$

$$= \frac{4x^2-1}{2x^2} + C$$

16. $\int (3x+1)^7 dx$

$$= \frac{(3x+1)^8}{8.3} + C = \frac{(3x+1)^8}{24} + C$$

17. $\int \cos(9x)dx$

$$= \frac{\sin(9x)}{9} + C$$

18. $\int (1 + tg^2x)dx$

$$= tgx + C$$

19. $\int 2\sin\frac{x}{5}\,dx$

$$= -2.\frac{1}{\frac{1}{5}}\,\text{cox}\,\frac{x}{5} + C = -10\cos\frac{x}{5} + C$$

20. $P = \int \frac{3x^2 + 2x - 1}{x - 1}\,dx$

Le degré du numérateur est supérieur au degré du dénominateur : on effectue une division polynômiale.

$$\begin{array}{r|l} 3x^2 + 2x - 1 & x - 1 \\ \hline 3x^2 - 3x & 3x + 5 \\ \hline 5x - 1 & \\ 5x - 5 & \\ \hline 4 & \end{array}$$

$$D(x) = d(x).q(x) + r$$

$$3x^2 + 2x - 1 = (x-1).(3x+5) + 4$$

$$\frac{3x^2 + 2x - 1}{x - 1} = (3x + 5) + \frac{4}{x - 1}$$

Donc $P = \int\left((3x+5) + \frac{4}{x-1}\right)dx$

$$= 3\frac{x^2}{2} + 5x + 4\ln|x - 1| + C$$

21. $\int (3x - 7)^6\,dx$

$$= \frac{(3x-7)^7}{7.3} + C = \frac{(3x-7)^7}{21} + C$$

22. $\int \frac{x^3 - 3x^2 + 1}{x^3}\,dx$

$$= \int\left(1 - 3\frac{1}{x} + x^{-3}\right)dx = x - 3\ln|x| + \frac{x^{-2}}{-2} + C = x - 3\ln|x| - \frac{1}{2x^2} + C$$

23. $\mathbf{P = \int \frac{2x^2 + 2x + 4}{x - 1} dx}$

$2x^2 + x + 4$	$2x - 1$
$\underline{2x^2 - x}$	$x + 1$
/ $\quad 2x + 4$	
$\quad \underline{2x - 1}$	
$\quad 5$	

$2x^2 + x + 4 = (2x - 1)\ (x + 1) + 5$

$$\frac{2x^2 + x + 4}{2x - 1} = x + 1 + \frac{5}{2x - 1}$$

$$P = \int \left(x + 1 + \frac{5}{2x - 1} \right) dx = \frac{x^2}{2} + x + 5 \frac{\ln|2x - 1|}{2} + C$$

$$\mathbf{= \frac{x^2}{2} + x + \frac{5}{2} \ln|2x - 1| + C}$$

24. $\mathbf{\int \frac{\sqrt[3]{x^4}.dx}{3}}$

$$= \frac{1}{3} \int x^{\frac{4}{3}} dx = \frac{1}{3} \frac{x^{\frac{7}{3}}}{\frac{7}{3}} + C = \frac{\sqrt[3]{x^7}}{7} + C = \frac{x^2.\sqrt[3]{x}}{7} + C$$

25. $\mathbf{\int e^{\pi.r} \left(r^3 + 7.r + \frac{14}{r^7} \right).dx}$

$$= \left(r^3 + 7.r + \frac{14}{r^7} \right).x + C$$